

Third Lecture: Polarized (helicity) PDFs

3.0 Helicity PDFs framework

3.1 Why DIS is not enough?

3.2 Why SIDIS is not enough?

3.3 Can pp $\rightarrow \pi^0$ /jets help?

3.4 Compass SIDIS update

3.5 RHIC preliminary/projections

3.6 EIC projections

3.7 DSSV gluon update (2014)

3.8 NNPDFs reweighting (2014)

3.9 Outlook

First NLO DIS analysis 1996

'90s, GRSV, AAC, BB, NNPDFs

First NLO DIS+SIDIS analysis 1998

DSS, DS, DNS, LSS

First NLO DIS+SIDIS+pp analysis 2008

DSSV

DSSV+

DSSV++

3.0 Helicity PDFs

factorization

universality

scale dependence

$$f_i^+(x,Q^2)$$
 same helicity as target $f_i(x,Q^2)$ opposite helicity

$$f_{i}(x,Q^{2}) \equiv f_{i}^{+}(x,Q^{2}) + f_{i}^{-}(x,Q^{2})$$

$$\Delta f_{i}(x,Q^{2}) \equiv f_{i}^{+}(x,Q^{2}) - f_{i}^{-}(x,Q^{2})$$

$$\Delta \sigma(ep \to eX) \equiv \sigma(\vec{e}p^{+} \to eX) - \sigma(\vec{e}p^{-} \to eX)$$

$$= \Delta \hat{\sigma}_{i} \otimes \Delta f_{i}$$

$$F_1(x,Q^2) = \frac{1}{2} \sum_q e_q^2 [q(x,Q^2) + \overline{q}(x,Q^2)] \longrightarrow g_1(x,Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x,Q^2) + \Delta \overline{q}(x,Q^2)]$$

3.1 Why DIS is not enough?

inclusive DIS gives

$$g_1^p(x,Q^2) g_1^n(x,Q^2)$$

unknowns

$$\begin{array}{c} \Delta u\,\Delta\overline{u}\,\Delta d\,\Delta\overline{d}\,\Delta s\,\Delta\overline{s}\,\Delta g\\ \text{assuming }\Delta u^p=\Delta d^n\,\text{etc.} \end{array}$$

evolution in principle could help:

$$\Delta q_{3}^{NS} \equiv (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}) \qquad \frac{d}{d \ln Q^{2}} \Delta q^{NS} = \frac{\alpha_{s}}{2\pi} \Delta P_{qq}^{1} \otimes \Delta q^{NS}$$

$$\Delta q_{8}^{NS} \equiv (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) - 2(\Delta s + \Delta \bar{s})$$

$$\Delta \Sigma \equiv (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s}) \qquad \frac{d}{d \ln Q^{2}} \left(\frac{\Delta \Sigma}{\Delta g}\right) = \frac{\alpha_{s}}{2\pi} \left(\frac{\Delta P_{qq}^{1}}{\Delta P_{gq}^{1}} \frac{2f P_{qg}^{1}}{P_{gg}^{1}}\right) \otimes \left(\frac{\Delta \Sigma}{\Delta g}\right)$$

$$g_{1}^{N}(x, Q^{2}) = \left(\pm \frac{1}{12} \Delta q_{3}^{NS} + \frac{1}{36} \Delta q_{8}^{NS} + \frac{1}{9} \Delta \Sigma\right) \otimes \left(1 + \frac{\alpha_{s}}{2\pi} \Delta C_{q}\right)$$

$$+ \sum_{q} e_{q}^{2} \frac{\alpha_{s}}{2\pi} \Delta g \otimes \Delta C_{g}.$$

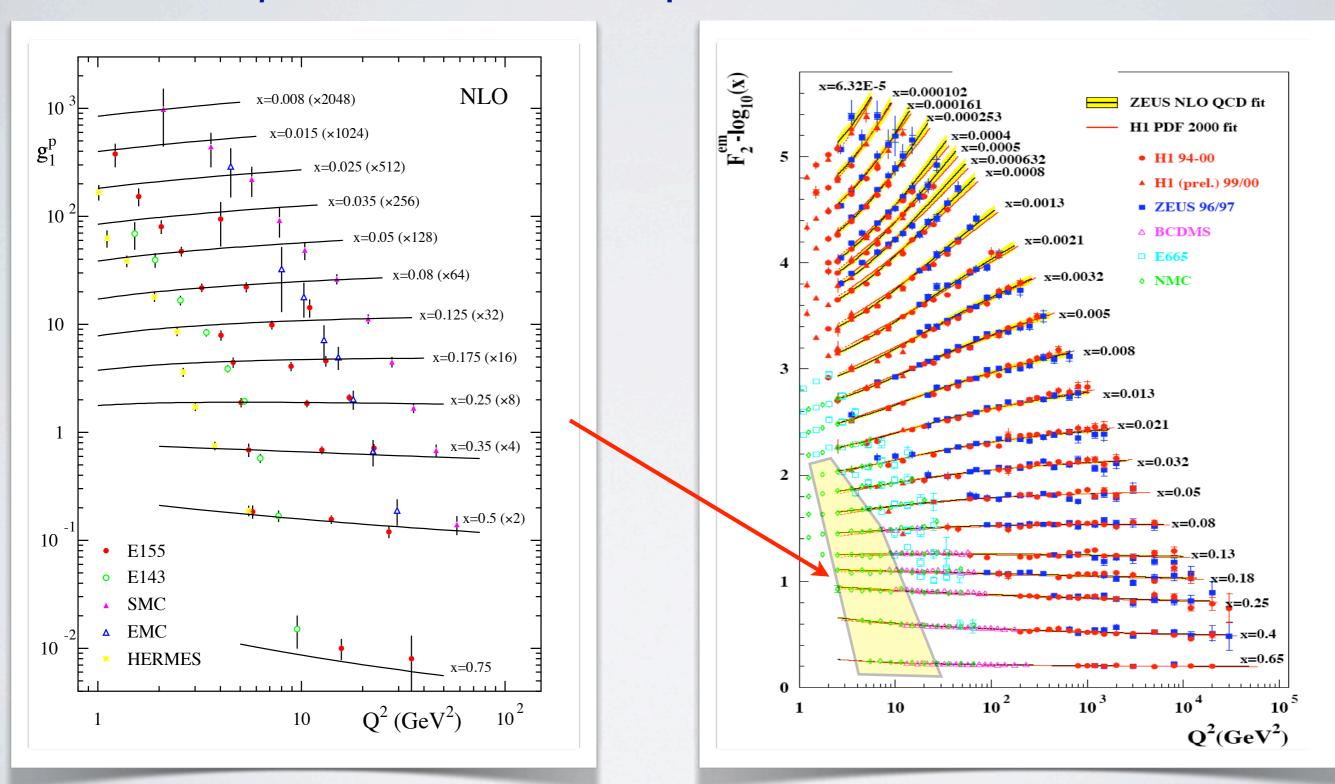
 $\Delta q_3^{NS}, \Delta q_8^{NS}, \Delta \Sigma, \Delta g \longrightarrow (\Delta u + \Delta \overline{u}), (\Delta d + \Delta \overline{d}), (\Delta s + \Delta \overline{s}), \Delta g$

 $\Delta u_v, \Delta d_v, \Delta \overline{q}, \Delta q$

wrong! SU(3)

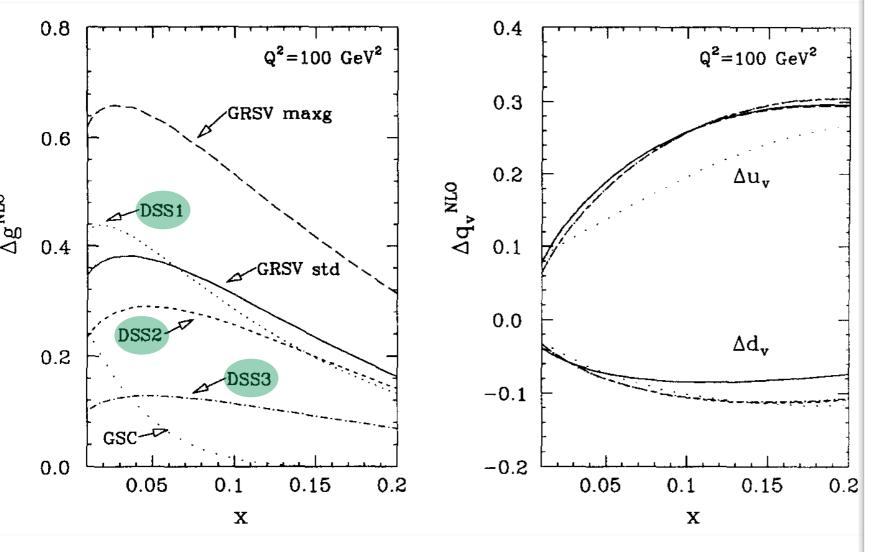
3.1 Why DIS is not enough?

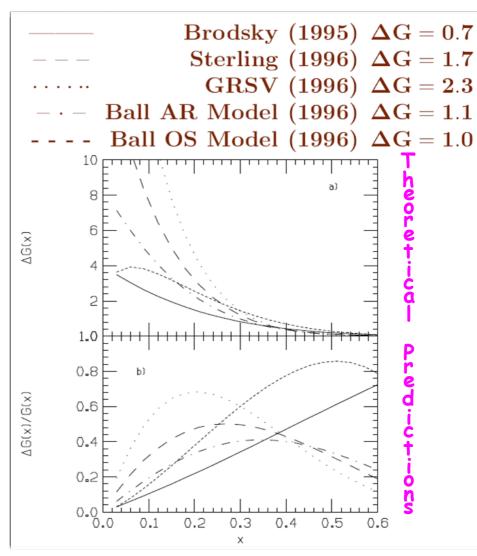
evolution in practice is of little help:



reduced kinematical coverage and precision compared to unpolarized case

3.1 Why DIS is not enough?





Not a surprise that different fits provide very different pdfs

same group, same data: three different gluons!

mea culpa

Similarly for first moment of gluon polarized pdf

3.2 Why SIDIS is not enough?

helps with sea quarks: analyzing power of final state hadrons

$$\begin{array}{ll} D_{1} = D_{u}^{\pi^{+}} = D_{\overline{d}}^{\pi^{+}} & 2g_{1p}^{\pi^{+}} & 2g_{1p}^{\pi^{+}}$$

but for a price: FFs dependence so far consistent -no visible tensions?needs independent check (W)
kaons??

huge exp. & Th errors

very little help with gluons: more precise $\Delta\Sigma$, better Δg same kinematics as DIS $O(\alpha_s)$ relative suppression charm and high pT hadrons

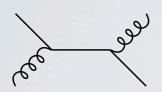
3.3 Can $pp \rightarrow \pi^0/jets$ help?

To measure gluon

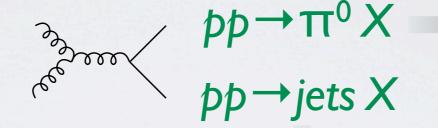


find observable where gluon starts at LO and dominates cross section

pp collisions: several processes initiated by gluons





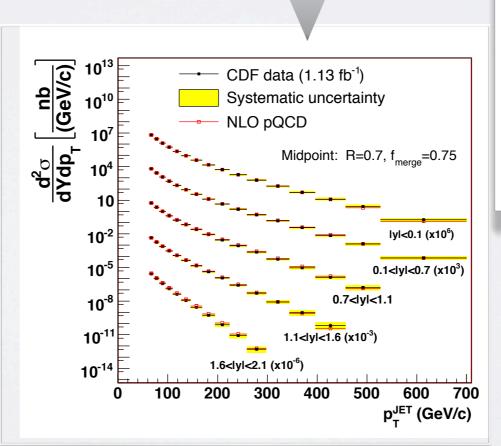


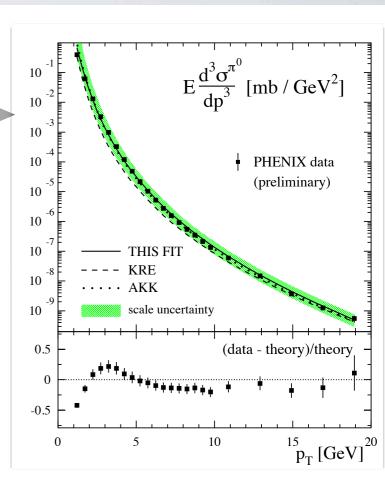
known NLO corrections V

work in unpolarized case 🗸

other interesting candidates:

$$\begin{array}{c} & & & \\ & &$$





3.3 Can $pp \rightarrow \pi^0/jets$ help?

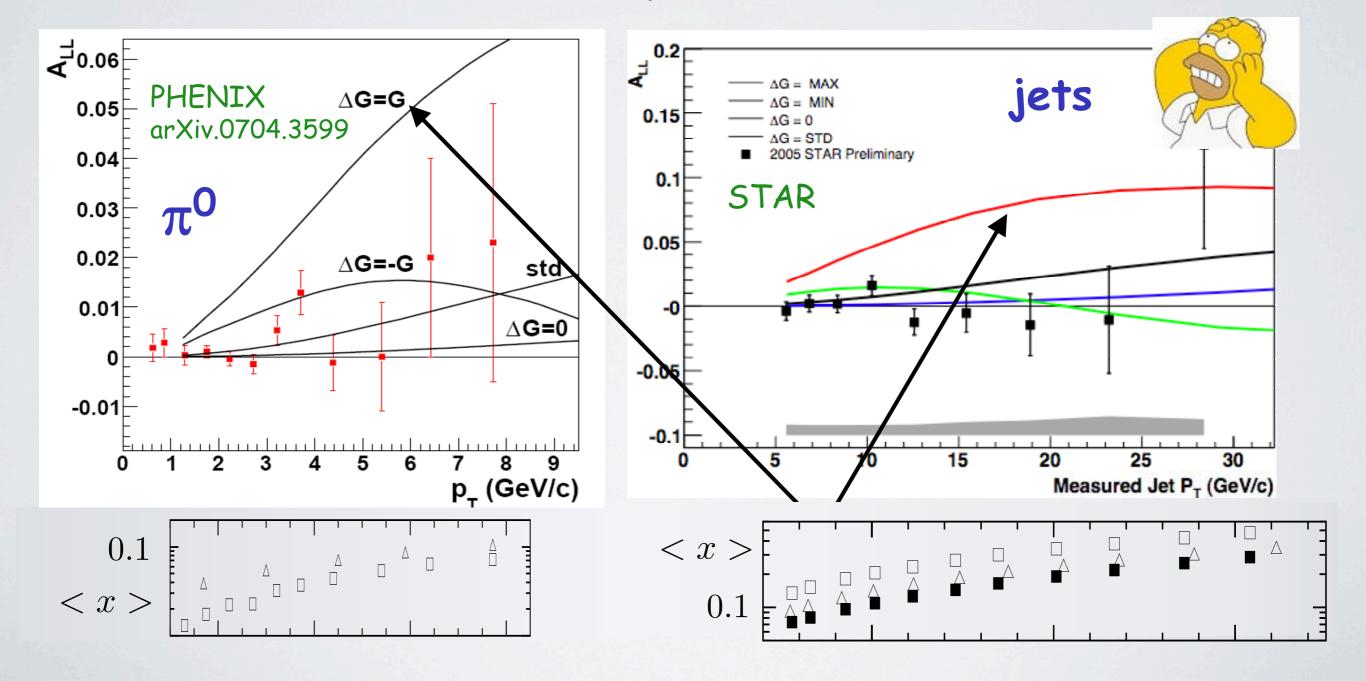
they have already done: ruled out large gluon polarization scenarios 🗸

however!: so small polarization that errors prevent other direct comparisons X

however²: kinematic entanglement

each p_T bin probes a different x different pieces of Δg p_T is the factorization scale

at different Q²



3.3 Can $pp \rightarrow \pi^0/jets$ help?

Double spin asymmetries

$$A_{LL} \equiv \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}} \equiv \frac{d\Delta\sigma}{d\sigma}$$

$$\pi^0$$
 jets ψ^{\pm} ψ^{\pm}

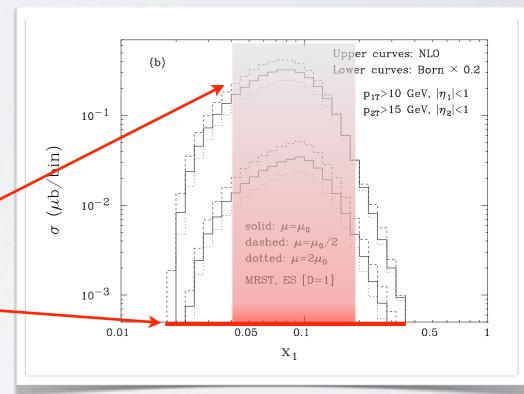
$$d\Delta\sigma \equiv \sum_{ab} \int dx_a \int dx_b \, \Delta f_a(x_a, Q^2) \Delta f_b(x_b, Q^2) \times d\Delta \hat{\sigma}_{ab}(x_a, x_b, p_T, \alpha_s(Q^2), p_T/Q)$$

$$x_a = (p_{aT}e^{\eta_a} + p_{bT}e^{\eta_b})/\sqrt{s}$$

$$x \sim 2p_T/\sqrt{S}$$

dominated by $x\sim0.04-0.2$

a range of x values



Single spin asymmetries : only for Parity violating interactions

$$A_L = -\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \qquad x_{a,b} = \frac{M_W}{\sqrt{s}} e^{\pm y_W}$$

what can we do?

DIS: no flavor separation, no gluons

SIDIS: no gluons, FF-dependence

pp: entanglement, limited range

global analysis

DSSV 2008 D.de Florian, R.S., M. Stratmann, W. Vogelsang		experiment	data type	data points fitted	χ^2
		EMC, SMC	DIS	34	25.7
inclusive DIS data $\Delta q + \Delta \overline{q}$	50%	COMPASS	DIS	15	8.1
		E142, E143, E154, E155	DIS	123	109.9
		HERMES	DIS	39	33.6
		HALL-A	DIS	3	0.2
		CLAS	DIS	20	8.5
SIDIS data ~ flavor separation		SMC	SIDIS, h^{\pm}	48	50.7
	40%	HERMES	SIDIS, h^{\pm}	54	38.8
			SIDIS, π^{\pm}	36	43.4
			SIDIS, K^{\pm}	27	15.4
		COMPASS	SIDIS, h^{\pm}	24	18.2
	1.00/	(1 1 /	$200\mathrm{GeV}\mathrm{pp},\pi^0$	20	21.3
RHIC data	10%	PHENIX (prel.)	$62\mathrm{GeV}\mathrm{pp},\pi^0$	5	3.1
		STAR (in part prel.)	200 GeV pp, jet	19	15.7
		TOTAL:		467	392.6

DSSV parameterization

Start evolution at $Q_0^2 = 1 \,\mathrm{GeV}^2$

MRST like input (easy to impose positivity)

For (better determined) $u_T=u+\bar{u}$, $d_T=d+\bar{d}$ unpolarized $x\Delta f_i(x,Q_0^2)=N_ix^{\alpha_j}(1-x)^{\beta_j}(1+\gamma_i\sqrt{x}+\eta_ix)$

For sea and gluons \bar{u} , \bar{d} , \bar{s} , g

$$x\Delta f_j(x,Q_0^2) = N_j x^{\alpha_j} (1-x)^{\beta_j} (1+\eta_j x)$$
 node allowed

Small x behavior of sea

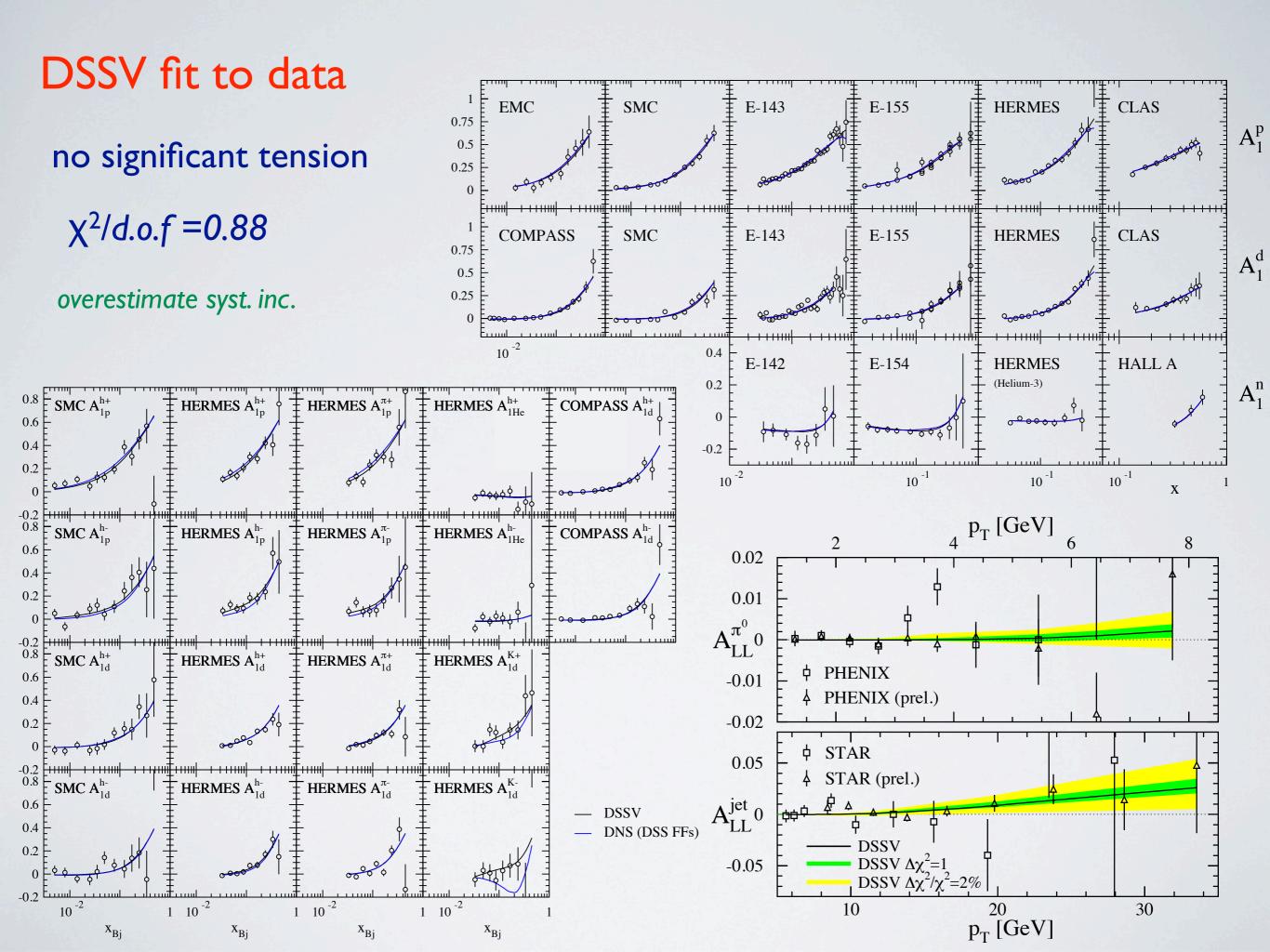
$$\alpha_{\bar{u}} = \alpha_{u_T} \qquad \alpha_{\bar{d}} = \alpha_{\bar{s}} = \alpha_{d_T}$$

26-5=21 free parameters

Normalization related to first moment

$$\Delta U_T - \Delta D_T = (F+D)[1+\varepsilon_{\mathrm{SU}(2)}]$$
 Allow deviations from standard assumptions
$$\Delta U_T + \Delta D_T - 2\Delta S_T = (3F-D)[1+\varepsilon_{\mathrm{SU}(3)}]$$

 $F + D = 1.269 \pm 0.003$, $3F - D = 0.586 \pm 0.031$



DSSV errors

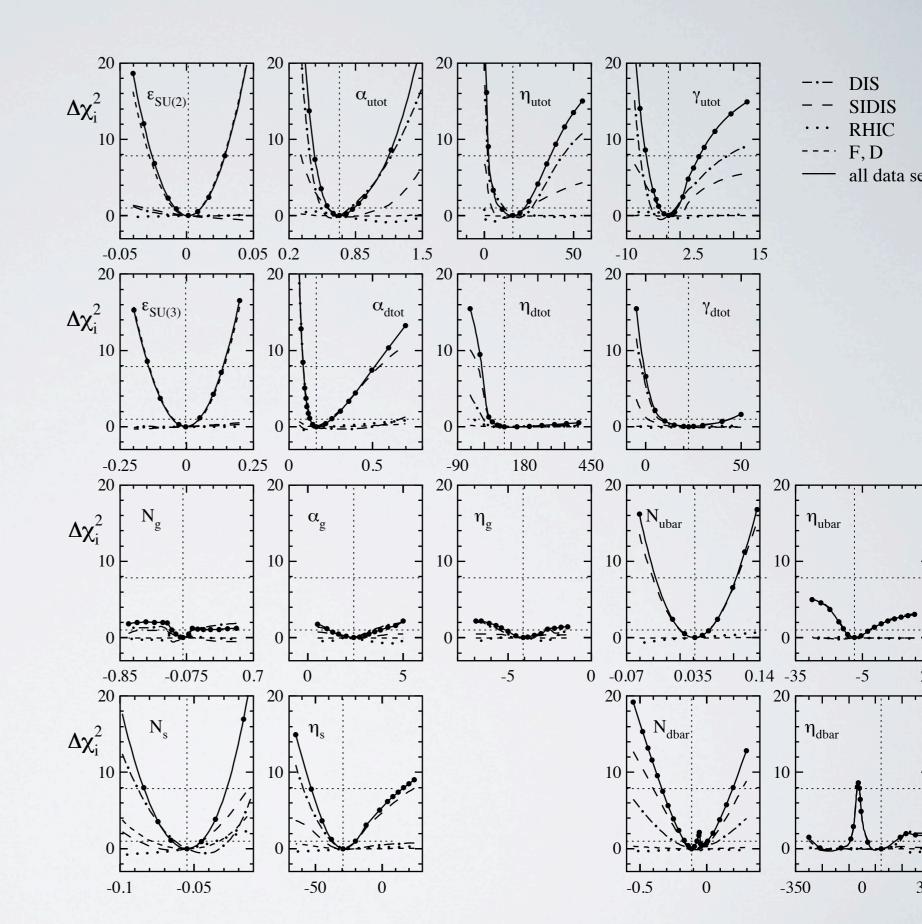
explore parameters profiles with LM

quadratic?

strong correlations between Δg parameters

non gaussian constraints:

- evolution
- positivity

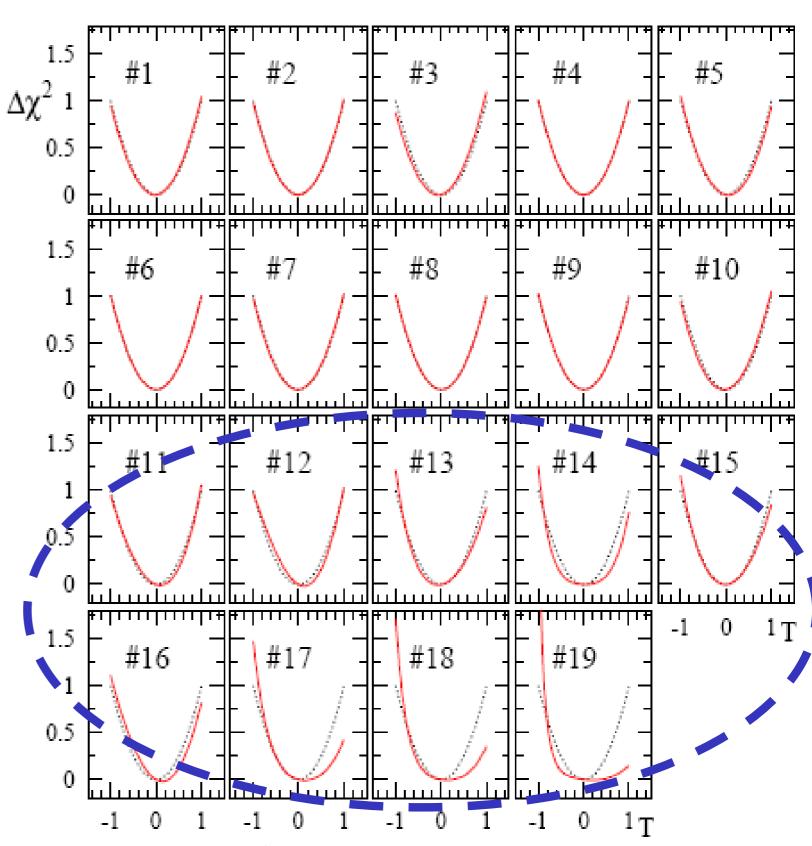


standard Hessian out of question

DSSV errors

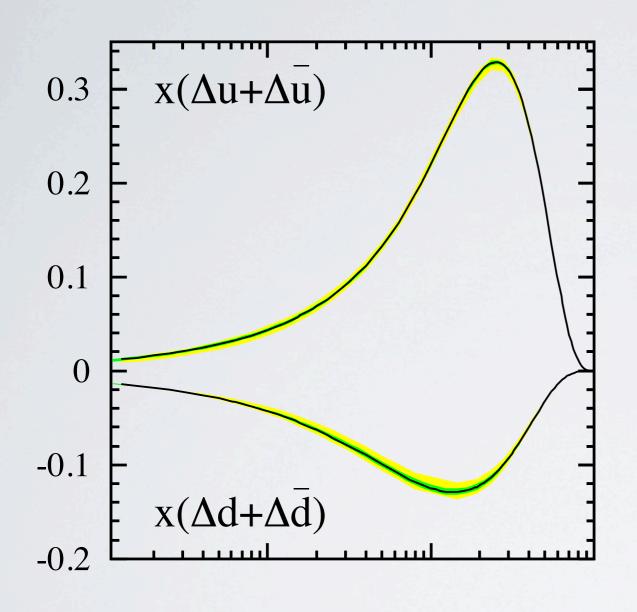
Improved Hessian also fails for $\Delta \chi^2 > 1$

 $\Delta \chi^2 = 1$ is insufficient



for a somewhat simplified DSSV fit with 19 parameters

DSSV PDFs



quark + antiquark pretty well determined : DIS

Green bands corresponds to $\Delta\chi^2=1$ Yellow bands corresponds to $\Delta\chi^2/\chi^2=2\%$

"Observable" for LM method: truncated moment [0.001 - 1]

$$\Delta f_j^{1,[x_{\min}-x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} \Delta f_j(x,Q^2) dx$$

~data available

DSSV sea quarks

0.02

-0.02

-0.04

 $Q^2 = 10 \text{ GeV}^2$

"Observable" for LM method: truncated moment [0.001 - 1]

$$\Delta f_j^{1,[x_{\min}-x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} \Delta f_j(x,Q^2) dx$$

~data available

		 		 	 		
0.04	_ x∆u		#	$x\Delta \bar{d}$		- 0.04	
			#				
0.02			1			$\frac{1}{2}$ 0.02	
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-0.02	_	10 10 10 10 10 10 10 10 10 10 10 10 10 1	1			-0.02	
	— DSSV		, 1			7	
-0.04	DNS	— DSSV	$\Delta \chi^2 = 1$			-0.04	
	GRSV	— DSSV	$\Delta \chi^2/\chi^2=2\%$				
		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1		10 -2	10 -1		
0.04	$- x\Delta s$			10	10	X	
0.01							

	$x_{\min} = 0$	$x_{\min} = 0.001$		
	best fit	$\Delta \chi^2 = 1$	$\Delta \chi^2/\chi^2 = 2\%$	
$\Delta u + \Delta \bar{u}$	0.813	$0.793 {}^{+0.011}_{-0.012}$	$0.793 {}^{+0.028}_{-0.034}$	
$\Delta d + \Delta \bar{d}$	-0.458	$-0.416 {}^{+0.011}_{-0.009}$	$-0.416 {}^{+0.035}_{-0.025}$	
$\Delta ar{u}$	0.036	$0.028 {}^{+0.021}_{-0.020}$	$0.028 {}^{+0.059}_{-0.059}$	
$\Delta ar{d}$	-0.115	$-0.089 ^{+0.029}_{-0.029}$	$-0.089 {}^{+0.090}_{-0.080}$	
$\Delta ar{s}$	-0.057	$-0.006 ^{+0.010}_{-0.012}$	$-0.006 {}^{+0.028}_{-0.031}$	
$\Delta\Sigma$	0.242	$0.366 {}^{+0.015}_{-0.018}$	$0.366 ^{+0.042}_{-0.062}$	

differences with previous fit (DNS): new DSS fragmentation functions

 $\Delta \bar{u}$ ~ positive

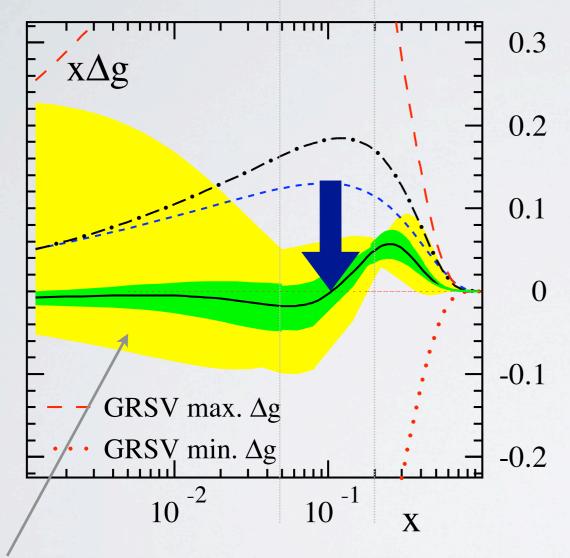
Robust pattern: SU(3) sea

 $\Delta ar{d}$: negative

 $\Delta \bar{s}$: SIDIS requires positive (HERMES) but first moment negative (DIS)

DSSV gluons

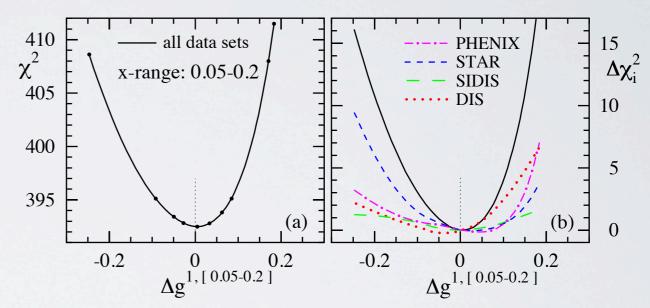
$$\Delta f_j^{1,[x_{\min}-x_{\max}]}(Q^2) \equiv \int_{x_{\min}}^{x_{\max}} \Delta f_j(x,Q^2) dx$$



no clear statement for first moment : ~0 but huge uncertainty at small x

Split in 3 regions

$$\begin{cases} 0.001-0.05 & \text{small x} \\ 0.05-0.2 & \text{'RHIC'} \\ 0.2-1 & \text{large x} \end{cases}$$



Complementarity of different data sets RHIC mainly in [0.05-0.2] region

--- DSSV
---- DNS --- DSSV
$$\Delta \chi^2 = 1$$

--- GRSV --- DSSV $\Delta \chi^2 / \chi^2 = 2\%$

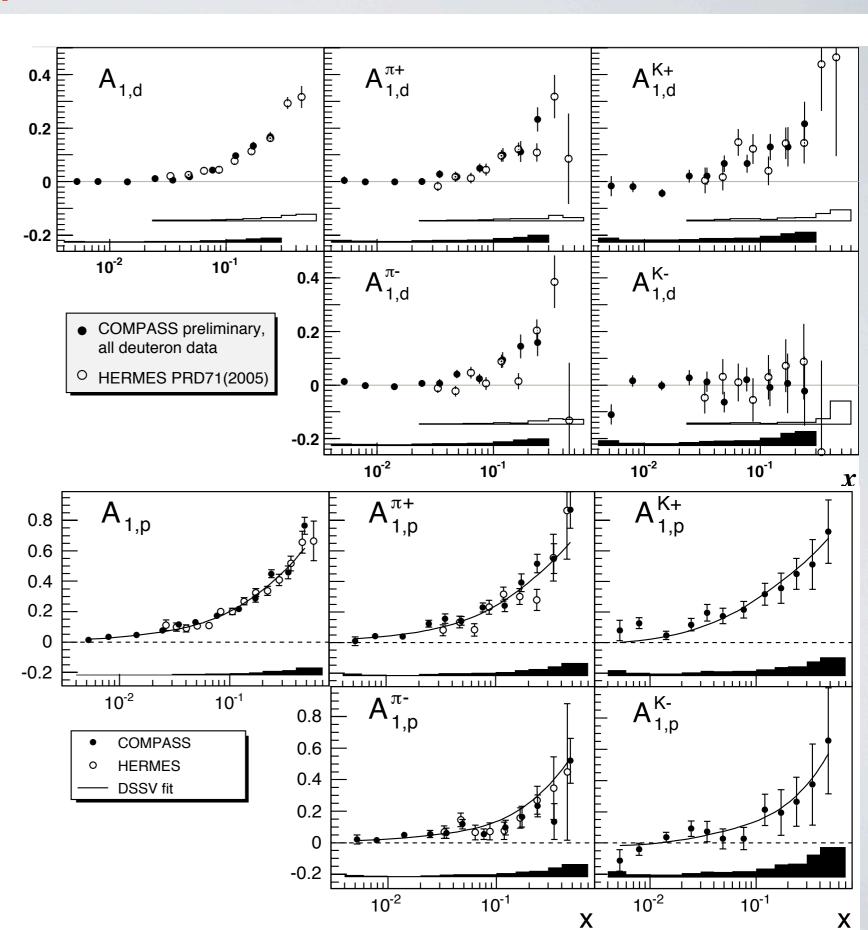
Δg rather small even moderate estimates (DNS/GRSV) ruled out

3.4 Compass SIDIS update: DSSV+

low x SIDIS $\mu+(p,d)\rightarrow(\pi,K)+X$ DIS at low-x DIS at high Q²

M.G. Alekseev et al., Phys.Lett.B680:217-224,2009.



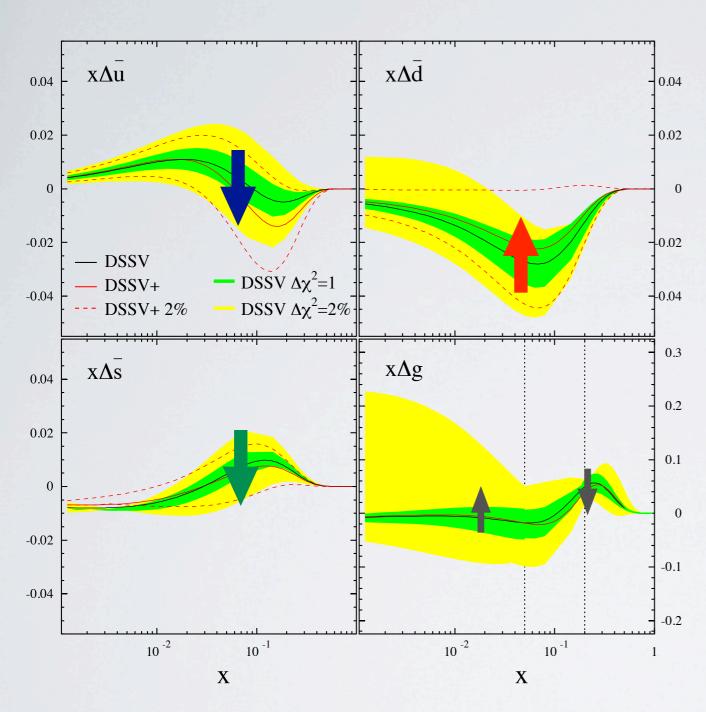


3.4 Compass SIDIS update: DSSV+

DSSV well within the error bars of DSSV+

$$\Delta \chi^2 = 1$$
 is too small $\Delta \chi^2 = 2\%$ is too much

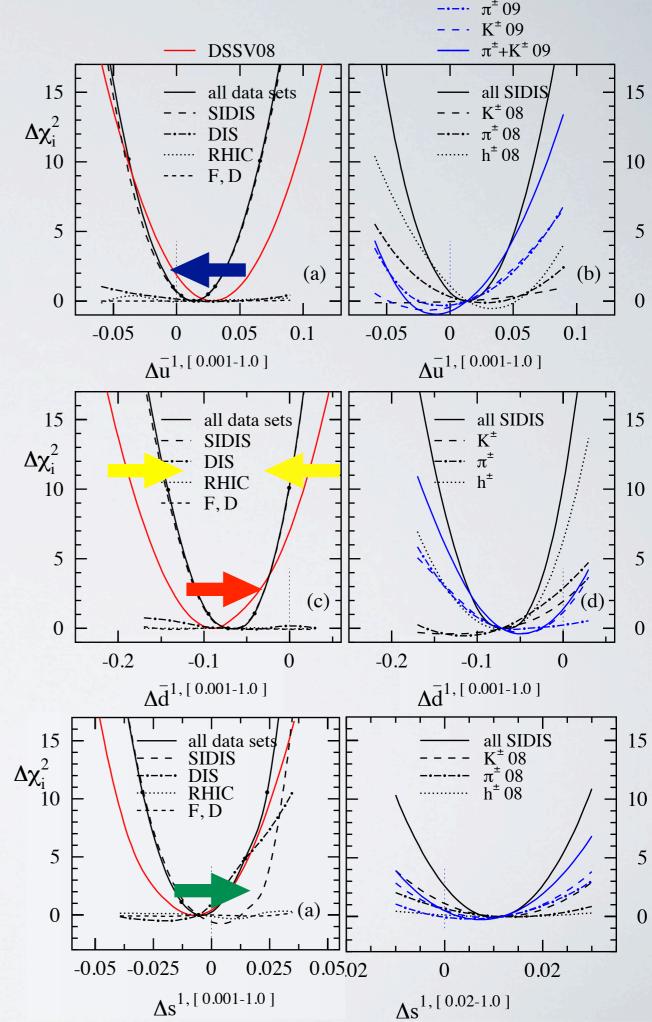
3.4 Compass SIDIS update:



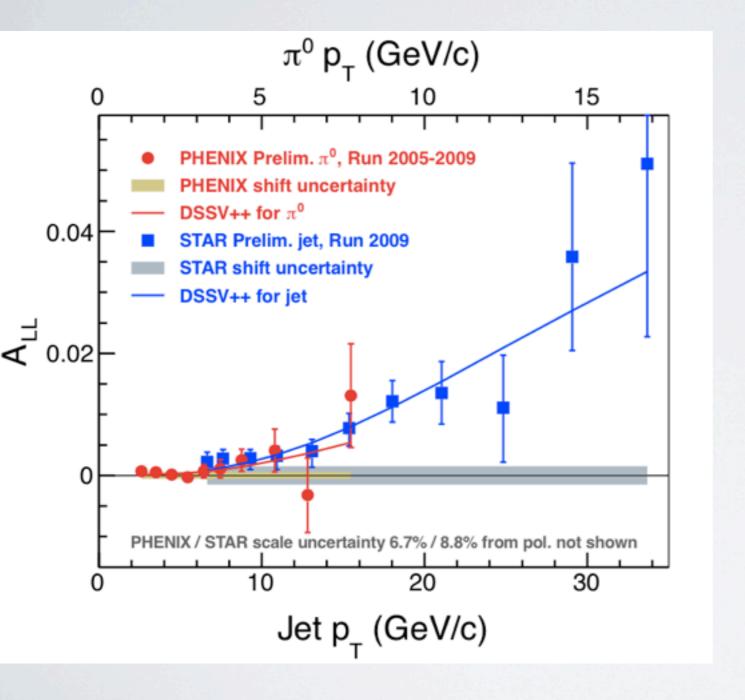
re-fit is well within DSSV bands

very minor shifts in moments some shrinkage in the bands

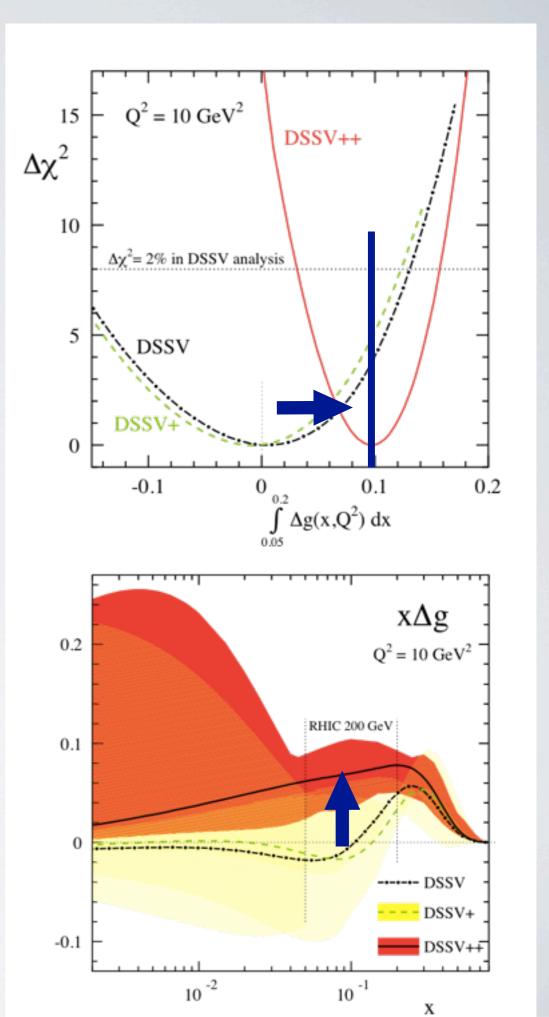
shifts in the minima of the parabolae reduction of width of the parabolae

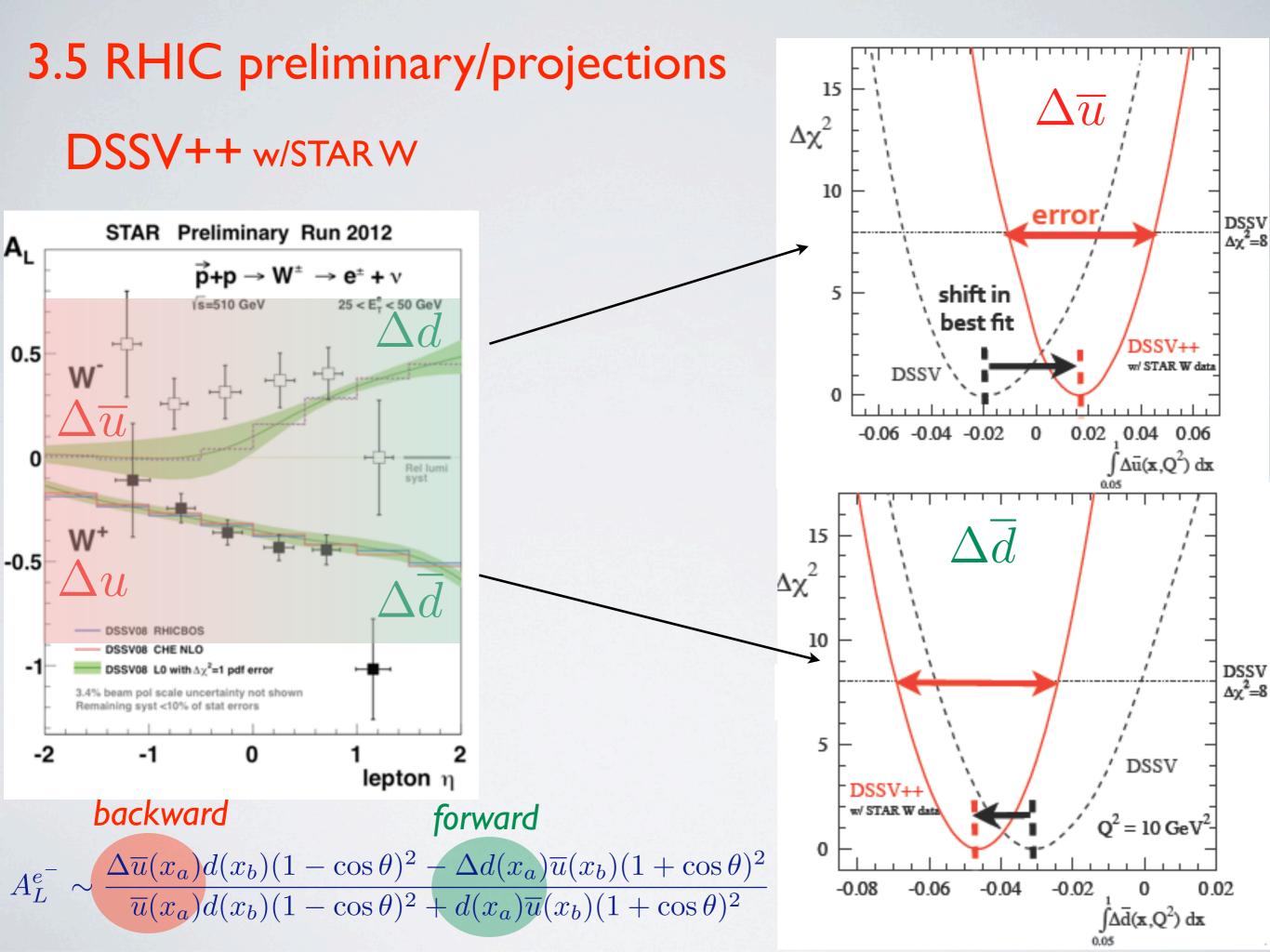


3.5 RHIC preliminary/projections

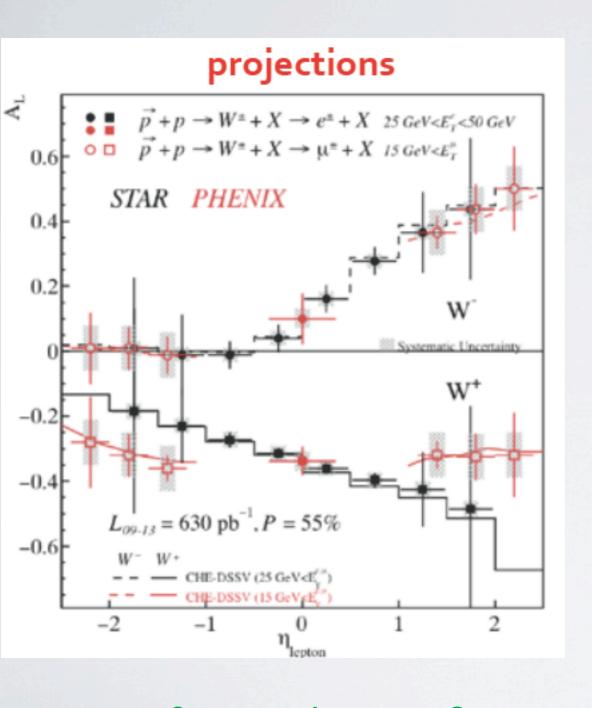


larger gluon polarization? (within errors)

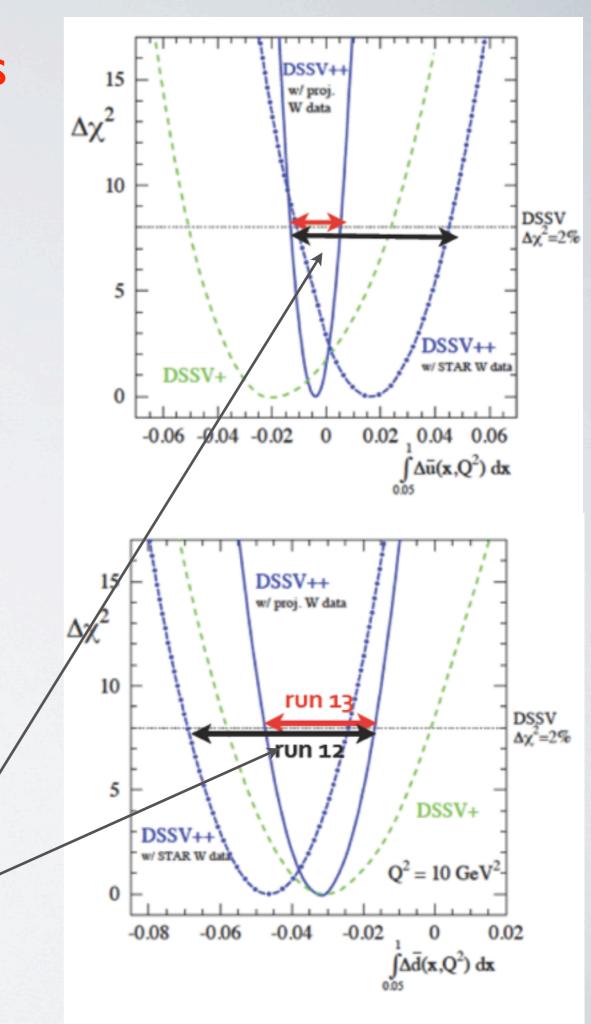




3.5 RHIC preliminary/projections



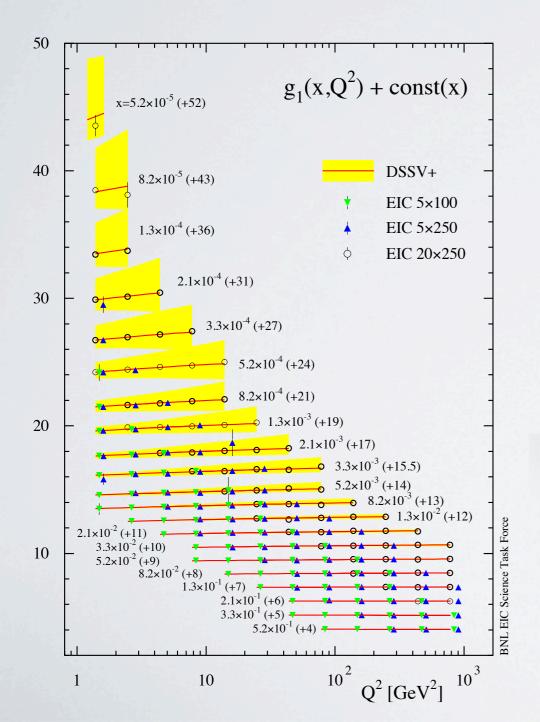
significant reduction of uncertainties
SIDIS & FFs cross check

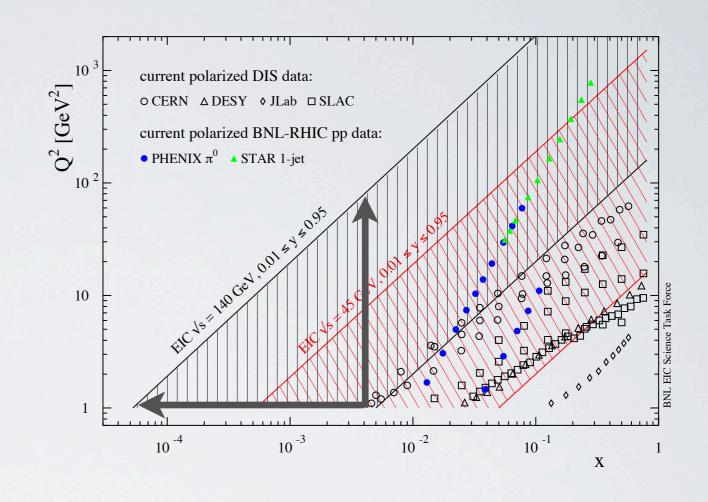


3.6 EIC projections Aschenauer, R.S., Stratmann 1209.3240

EIC kinematic reach:

- two decades in x
- large Q² coverage

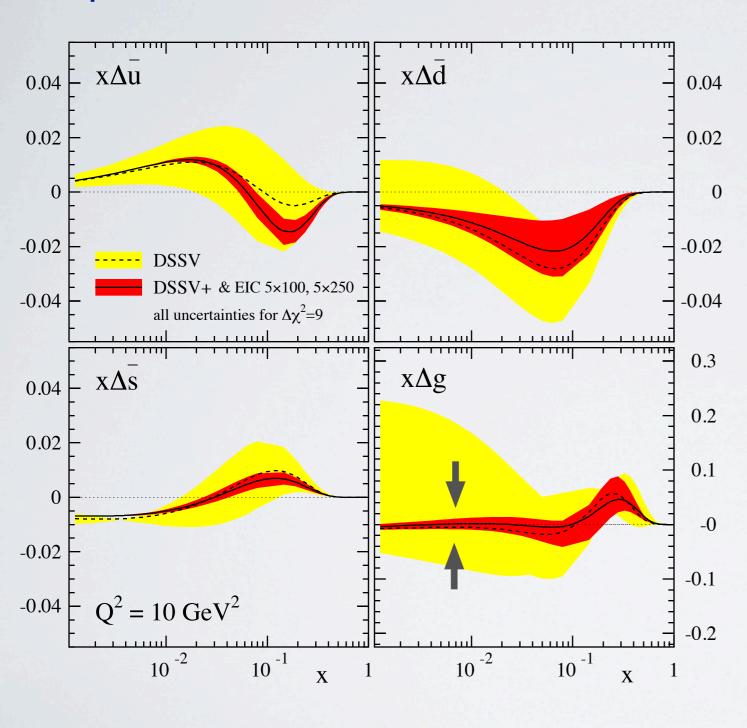


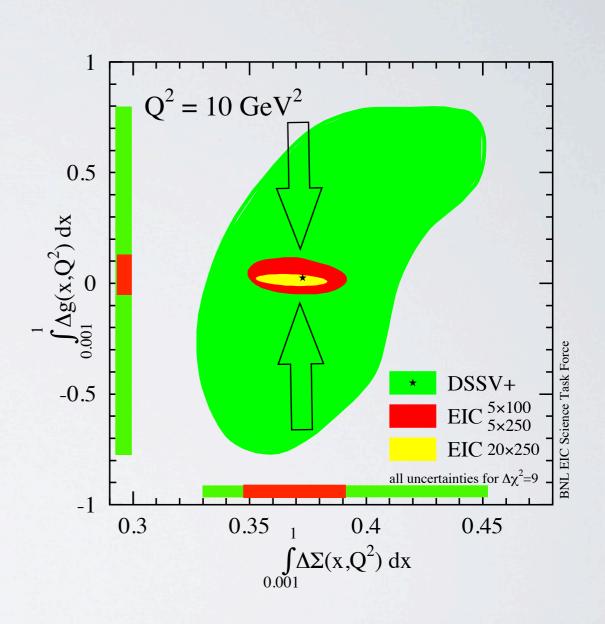


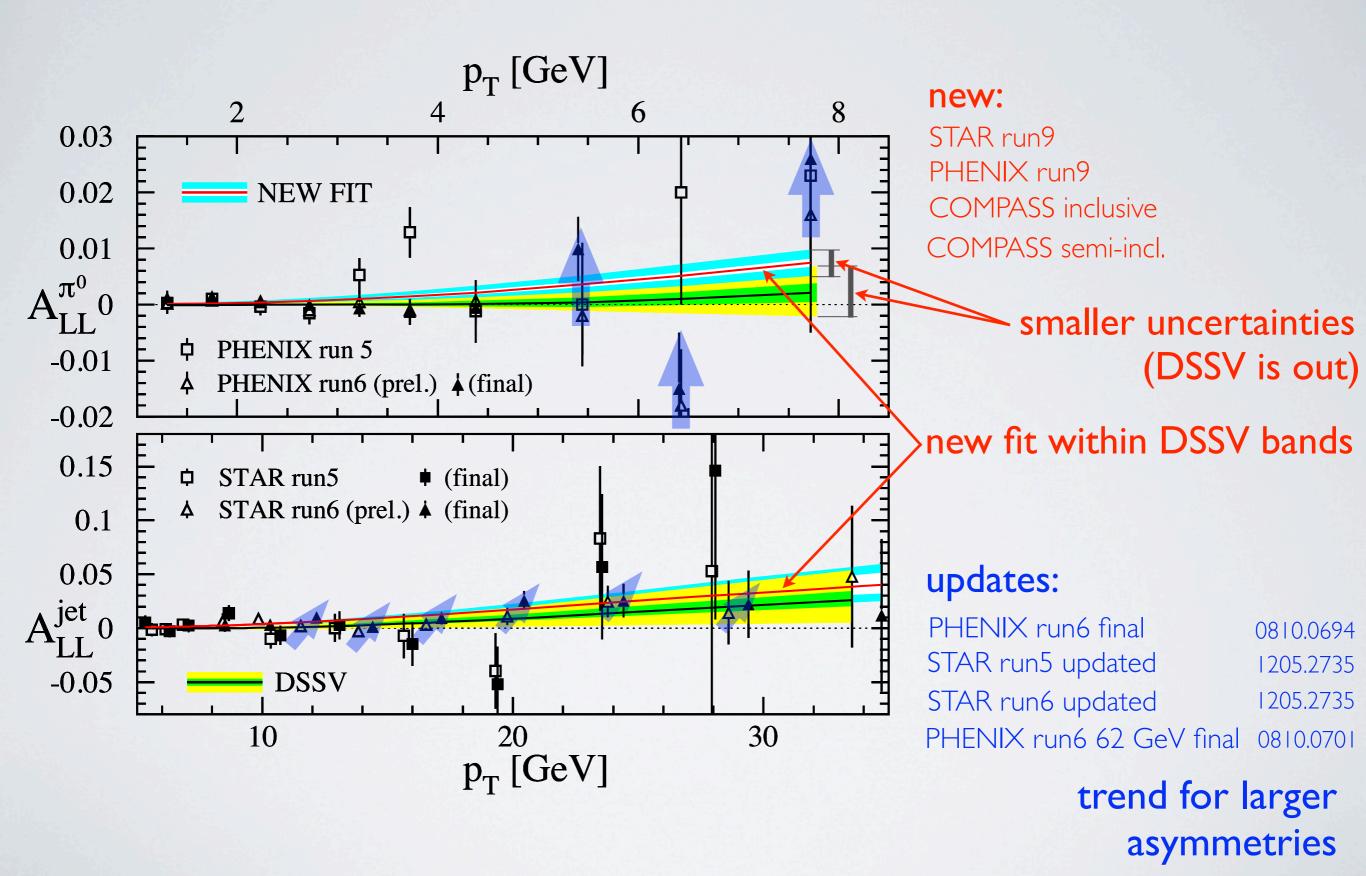
- precise DIS & SIDIS
- low-x
- Δg from scaling violations
- charged current DIS

3.6 EIC projections Aschenauer, R.S., Stratmann 1209.3240

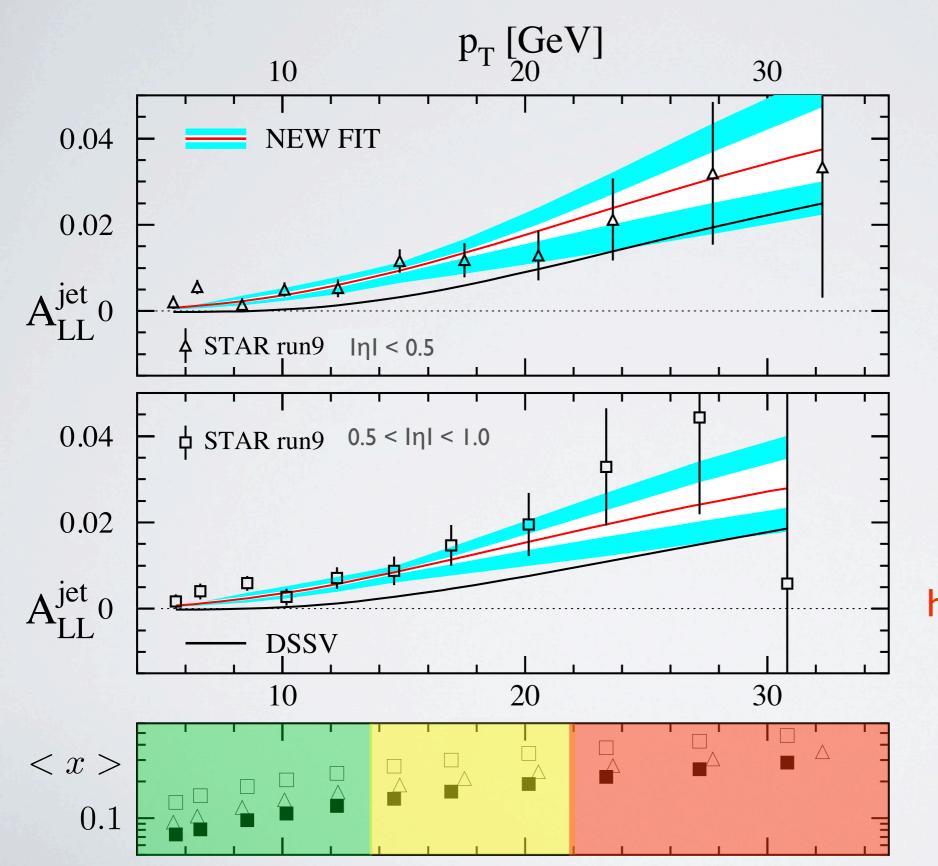
Impact from DIS & SIDIS







anti-k_⊤ jets



$$d_{ij} = min(p_{T,i}^{2p}, p_{T,j}^{2p}) \frac{\Delta_{ij}^{2}}{R^{2}}$$

$$\Delta_{ij}^{2} = (y_{i} - y_{j})^{2} + (\phi_{i} - \phi_{j})^{2}$$

$$d_{iB} = p_{T,i}^{2p}$$

two rapidity ranges

$$|\eta| < 0.5$$

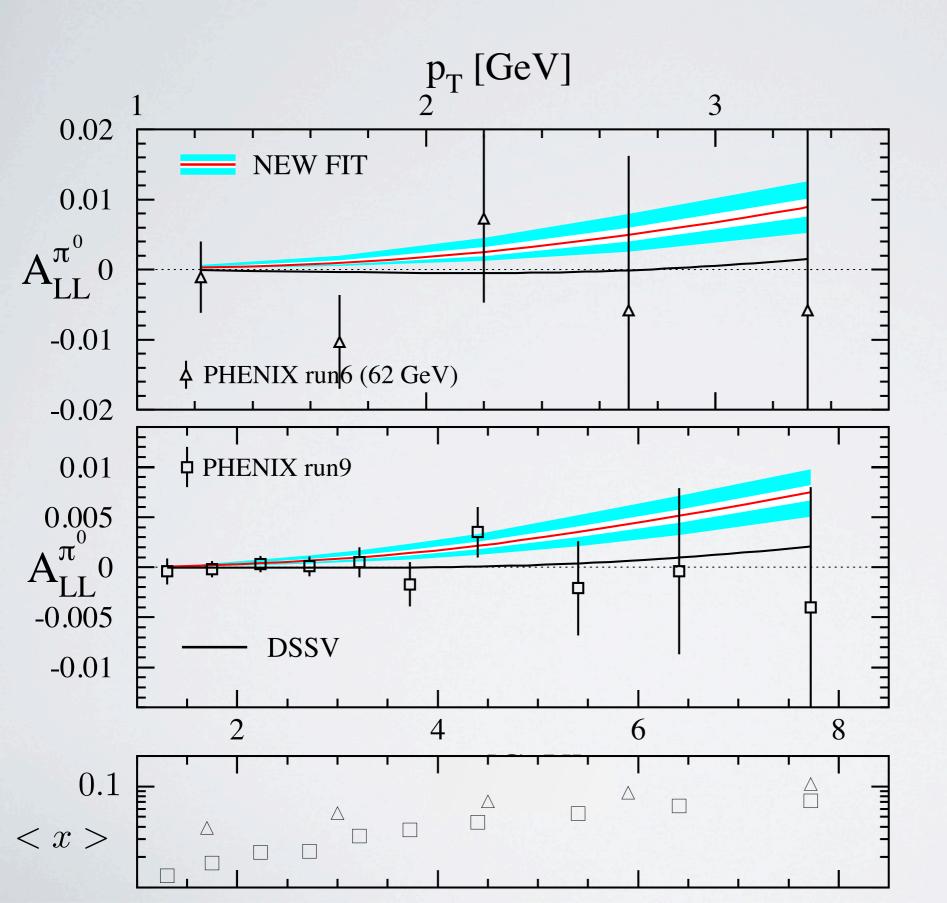
 $0.5 < |\eta| < 1.0$



higher precision (scale X3)



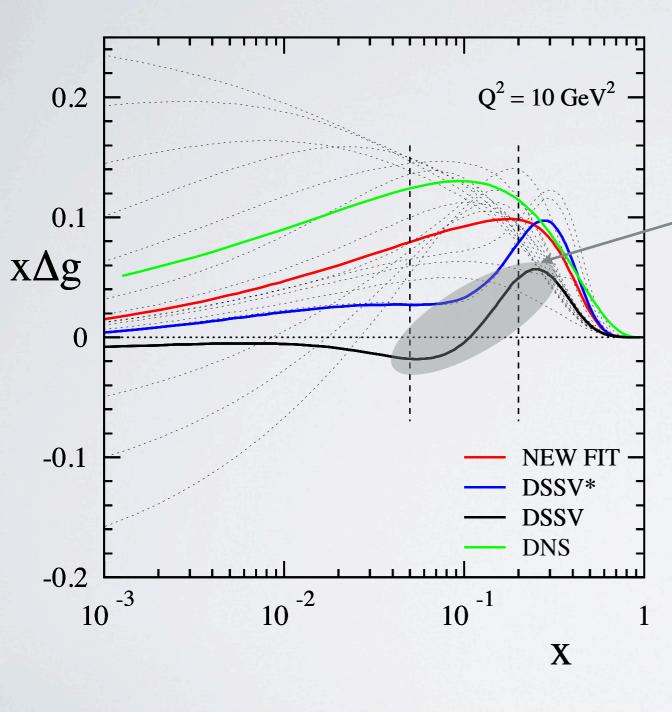
above DSSV

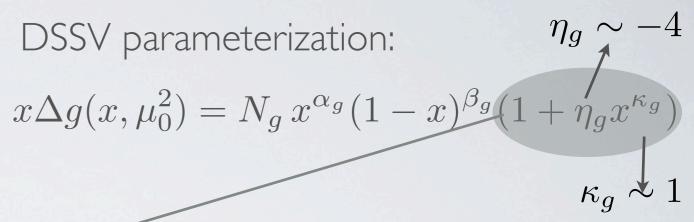


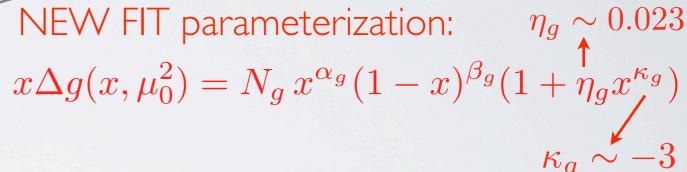
prefer DSSV?

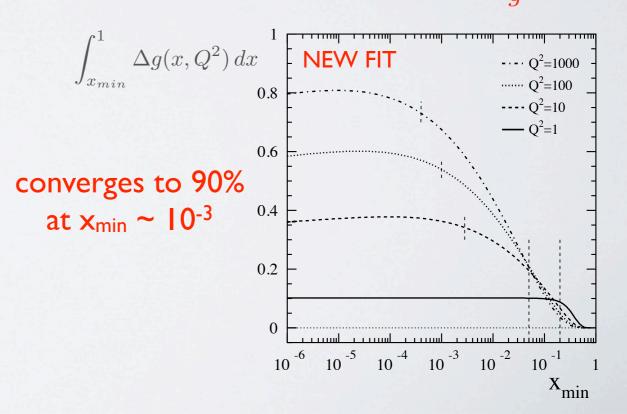
negative helicity?

The gluons



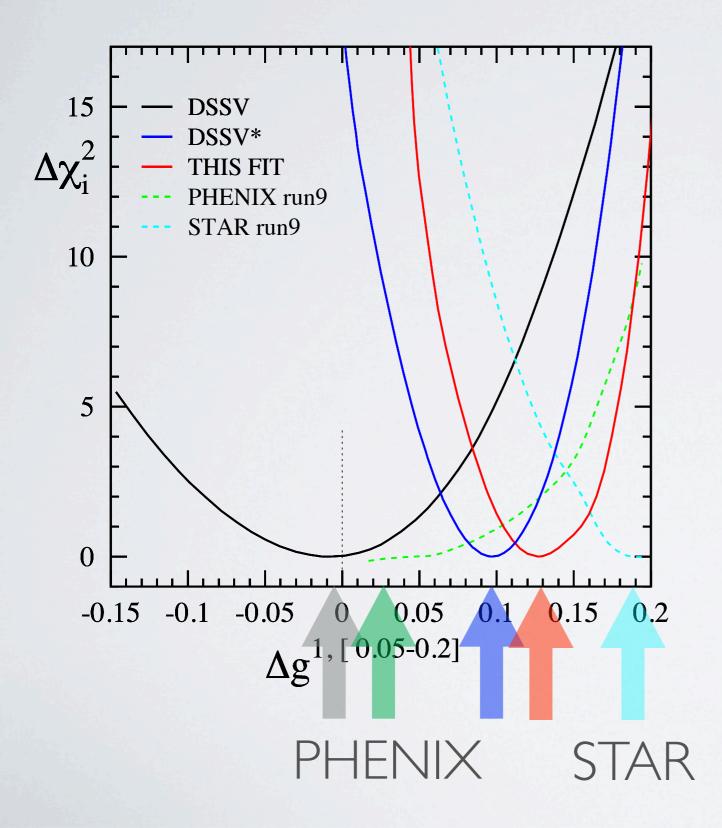






3.7 DSSV gluon update

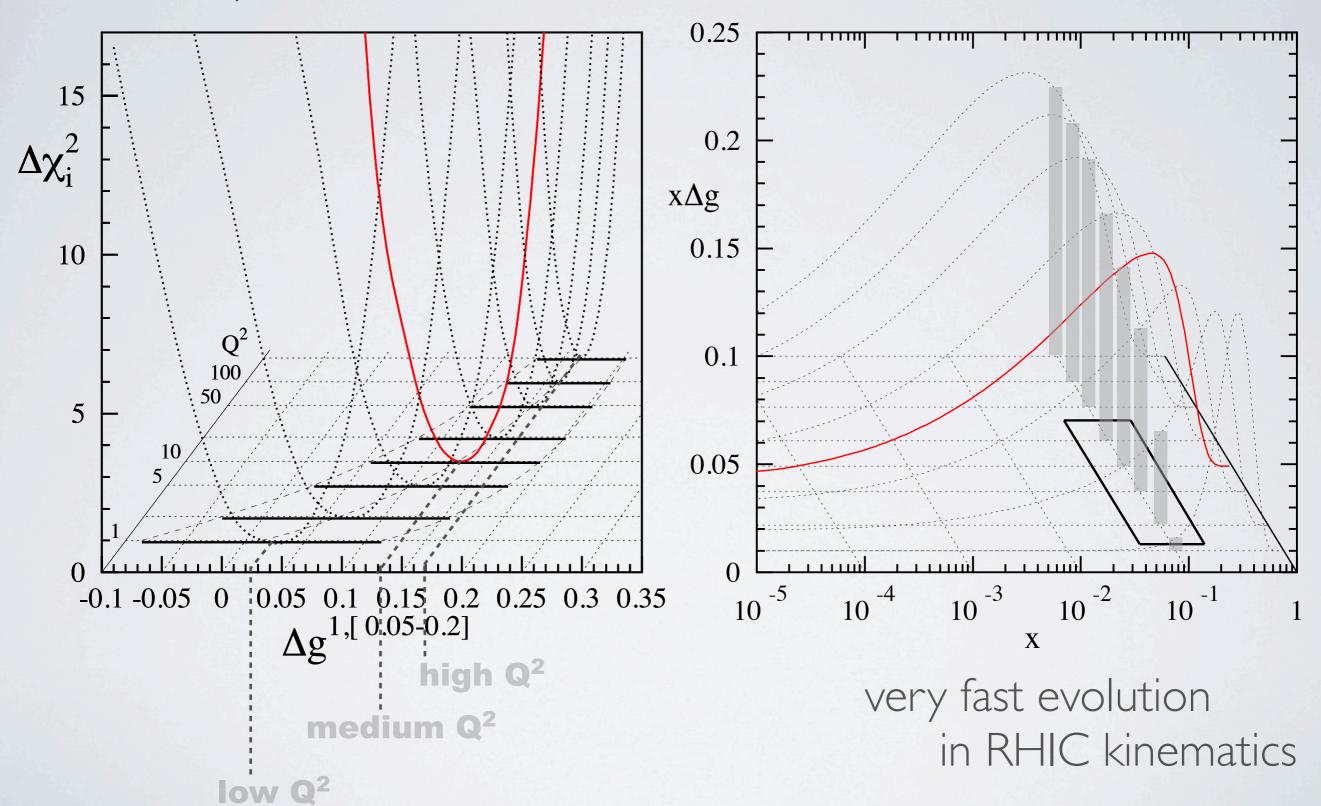
Truncated moments



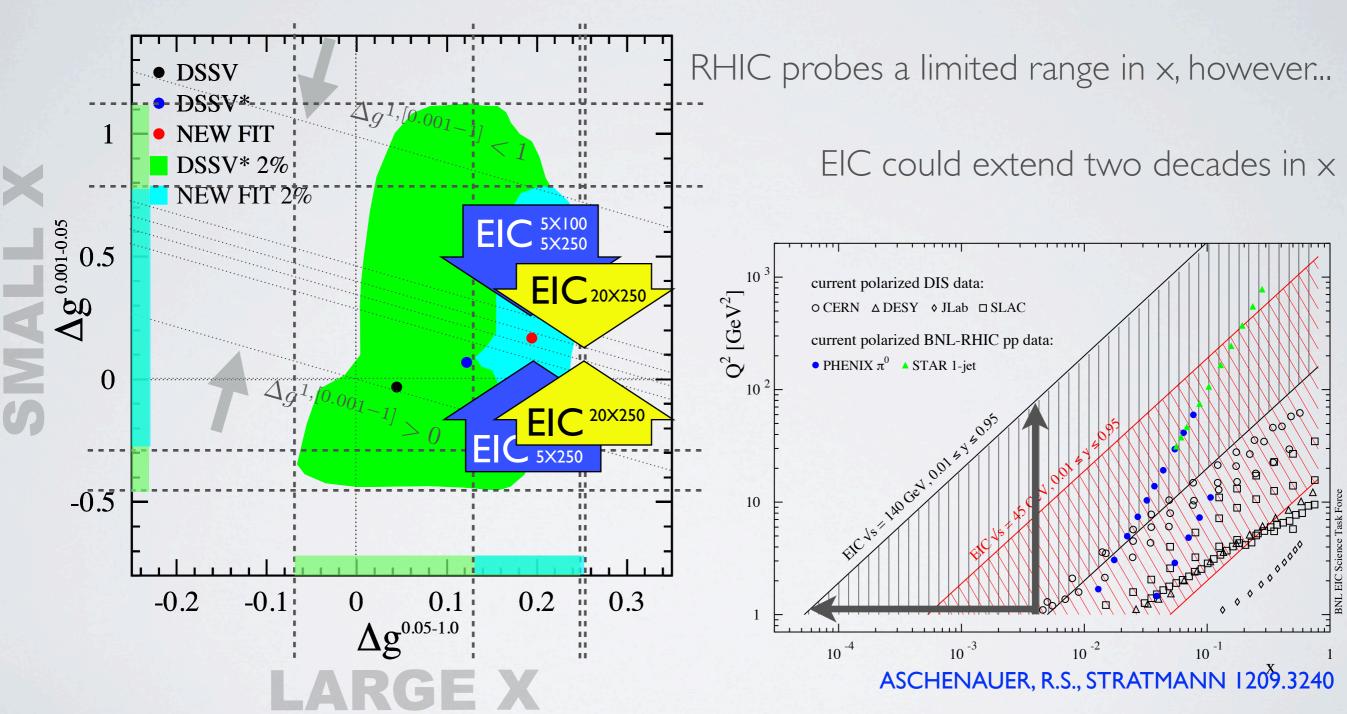
$$\Delta g^{1,[0.05-0.2]}(Q^2) \equiv \int_{0.05}^{0.2} \Delta g(x,Q^2) dx$$

tension?

complementarity: Q²-dependence Q²-dependence



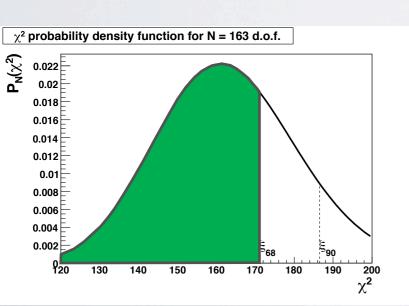
what about small and large-x?



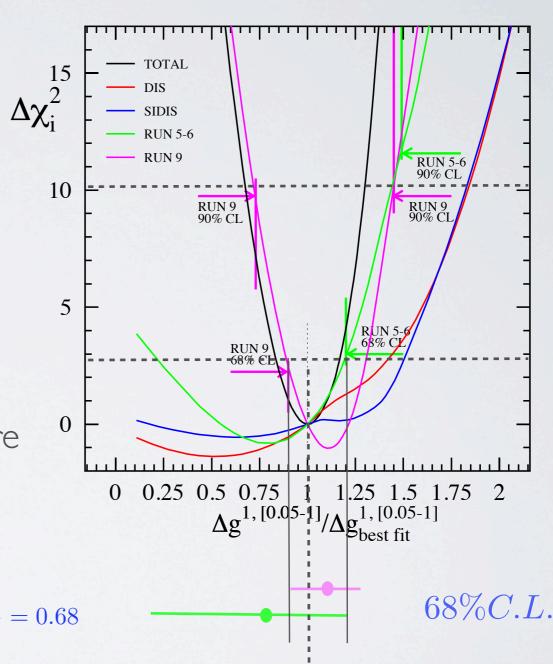
how do we estimate errors?

can't use MSTW & CTEQ dynamical scheme ~ Improved Hessian

- I. pick a relevant observable (not a parameter)~ truncated moment
- 2. plot profiles for subsets of data ~ experiment, data set
- 3. check their own 68% and 90% CL limits \sim assume χ^2 distribution
- 4. read error and tolerance for largest departure



$$\int_0^{\xi_{68}} d\chi^2 \, \frac{(\chi^2)^{N/2 - 1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)} = 0.68$$



3.8 NNPDF reweighting

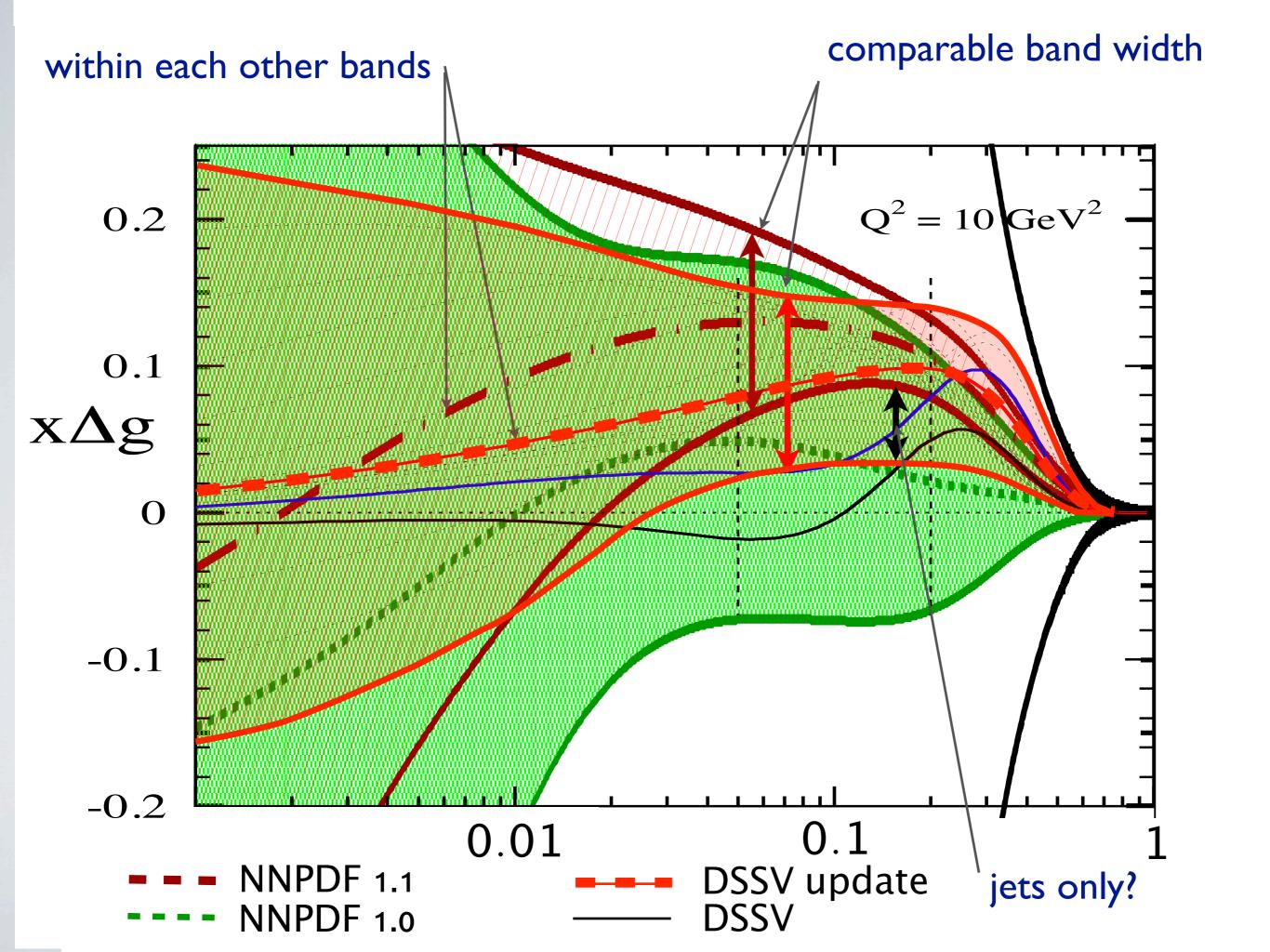
E. Nocera et al. 1406.5539

include RHIC (jets and W's) and Compass open charm data by reweighting build and ensemble of PDFs and prune by comparison with new data

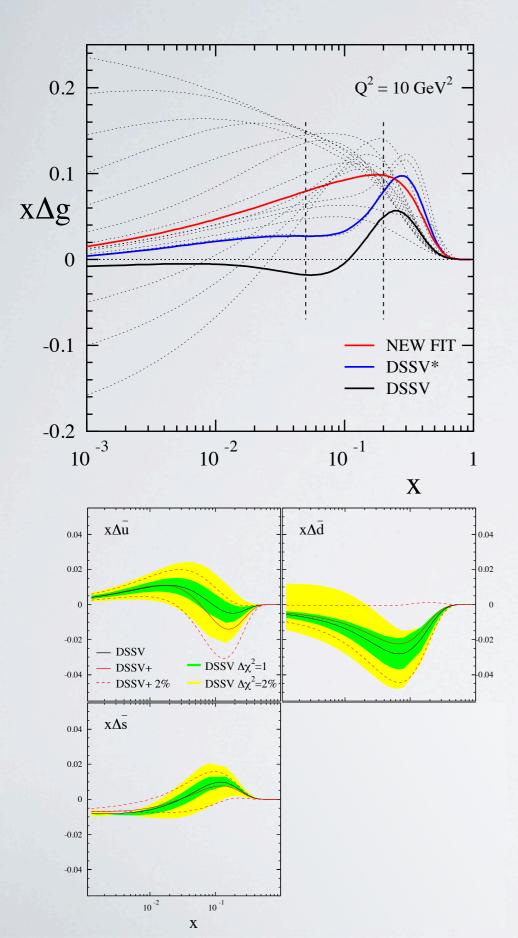
Problem: NNPDF1.0 produce only ensembles for $(\Delta q + \Delta \overline{q})$

Solution: borrow DSSV $\Delta \overline{q}$ but inflating $\Delta \chi^2 = 1$ parameter errors

- Compass open charm has negligible impact.
- W-data produce similar effect as in DSSV++ studies
- Good agreement on the gluon polarization with DSSV gluon update



3.9 Outlook



GLUONS:

keep giving us surprises ... nice: large!
much better constrained by data well done!
progress understanding uncertainties independent approaches

low-x: EIC obvious, but in long run

forward inclusive hadrons very welcome!

SEA QUARKS:

much interesting than $\Delta \overline{u} = \Delta \overline{d} = \Delta \overline{s} = 0$ lots of physics involved!

flavor separation: FFs upgrade almost done! Ws framework running







